COUPLED INDUCTORS IMPROVE MULTIPHASE BUCK EFFICIENCY

Introduction

Multiphase buck regulators have greatly reduced output ripple and response times compared to single-phase implementations. Using coupled inductors further reduces the ripple in each phase.

There has been considerable discussion within the power supply industry about the operation and benefits of using coupled inductors as the magnetic components for multiphase buck regulators supplying power for processors in desktop, notebook and server applications. But, how do the coupled inductors compare with the standard noncoupled multiphase and single-phase topologies?

Single-Phase Buck Regulator

A simplified schematic for a single-phase buck regulator is shown in **Fig. 1a.** During state one, the input voltage is connected to the inductor (S1H closed, S1L open), and energy is both stored in the inductor and transferred through the inductor (L) to the output. During state two, the input voltage is disconnected from the circuit (S1H open, S1L closed), the inductor is tied to ground and the stored energy in the inductor is transferred to the output. This sequence is repeated at a certain time interval (T_S) and the inverse of this interval is known as the switching frequency ($F_S = 1/T_S$). The ratio of the on time (T_{ON}) to the switching interval (T_S) is known as the duty cycle ($D = T_{ON}/T_S$). In a buck regulator, the output voltage is set by the duty cycle ($V_{OUT} = DV_{IN}$).

During state one, the on time (from t=0 to t=DT_S), the schematic reduces to **Fig. 1b.** It is clear from this figure that during the on time the voltage across the inductor (V_L) is equal to $V_{IN} – V_{OUT}$. From Faraday's law, we know that voltage across an inductor is equal to the inductance (L) times the rate of current change ($V_L = L di/dt$), and therefore:

$$di_1 = di_{OUT} = (V_{IN} – V_{OUT})\,(dt/L)$$

but $V_{OUT} = DV_{IN}$ and in state one $dt = DT_S$, therefore:

(Eq. 1) $$di_1 = di_{OUT} = (V_{IN}/L)\,(1 – D)\,DT_S$$

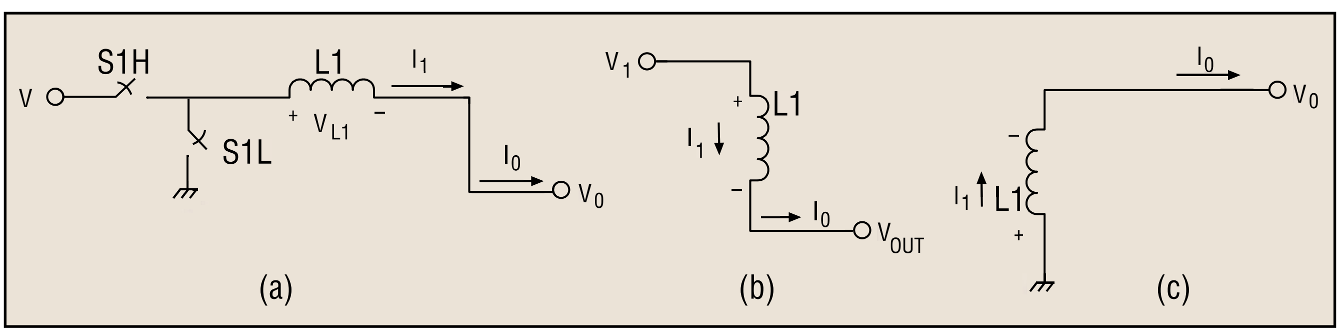

Fig. 1. A simplified schematic of a single-phase buck regulator (a) illustrates its two states of operation. In state one, S1H is closed and S1L is open, so that the input sources energy to the output and L1 stores energy (b). In state two, S1L is closed and S1H is open, so that L1 sources energy to the load (c).

From Eq. 1, it can be seen that during state one, the current through the inductor is increasing because energy is being stored in the inductor from the input.

During state two, the off time (from $t=DT_S$ to T_S), the schematic reduces to **Fig. 1c**, which shows that during the off time the voltage across the inductor is $–V_{OUT}$, and again using Faraday's law, we have:

$$di_1 = di_{OUT} = (–V_{OUT})(dt / L).$$

But $V_{OUT} = DV_{IN}$

and in state two $dt = T_S – DT_S = (1–D)T_S$

(Eq. 2) $$di_1 = di_{OUT} = (–V_{IN}/L)(1 – D)DT_S$$

From Eq. 2, it can be seen that during state two the current is decreasing because energy is being sourced to the output from the inductor. Note that the increase in current in state one is equal to the decrease in current during state two. It is clear that the ripple current (di_1) through the inductor is the same as the output ripple current (di_{OUT}).

Multiphase Buck Regulator

There are three main limitations of the single-phase buck regulator if employed in a voltage regulator for desktop, notebook or server applications. First, the high currents — greater than 40 A for notebook, 120 A for desktop and 150 A for server — would cause excessive I^2R losses if delivered over one path or phase. Second, processors require low- output ripple voltage and this necessitates keeping the output ripple current low, as $V_{RIPPLE} = I_{RIPPLE}$ ESR. This implies the need for a large inductor because I_{RIPPLE} is proportional to 1/L.

Third, the processor power supply must be able to respond quickly to changes in power requirements. Unfortunately, the third requirement, fast transient response, implies the need for a small inductor to allow the current through the supply to change quickly, and this conflicts directly with the need for a larger inductor to minimize output voltage ripple.

The uncoupled multiphase buck regulator was designed to resolve these three limitations. Instead of using a single high-current path, the multiphase buck

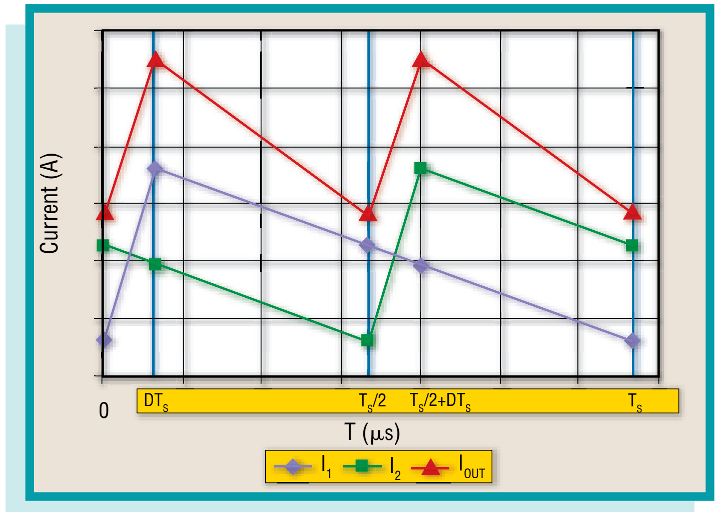

Fig. 3. In the uncoupled two-phase buck, the phase currents are out of phase. In the coupled two-phase buck, as the coupling ratio is reduced to zero, the waveforms of the two regulators become identical, suggesting that the uncoupled inductor can be treated as a special case of the coupled inductor solution.

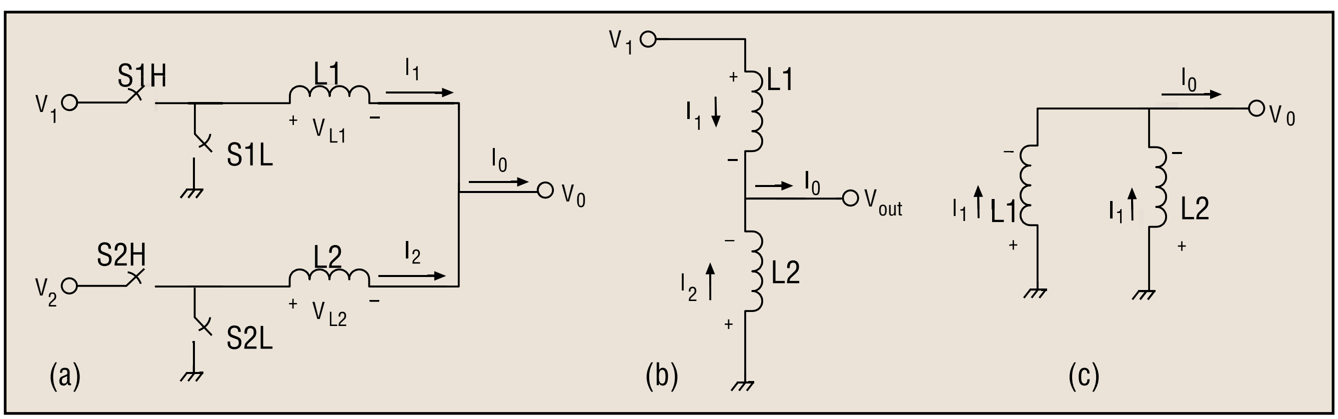

Fig. 2. A simplified schematic of a multiphase uncoupled buck regulator (a) illustrates the two basic switching actions. In state one (b), S1H and S2L are closed while S1L and S2H are open. The input then sources energy to L1 and the output, and L2 sources energy to the output. In state two (c), S1L and S2L are closed, and S1H and S2H are open. Thus, both inductors source energy to the output. These operations are reversed for states three and four (not shown).

breaks the current into several lower current parallel paths or phases. Each phase has its own inductor and set of switches, and the current in each phase is summed to form the output current. By activating each phase at a different point in the cycle, the ripple currents of each phase can be overlapped to reduce the overall output current ripple. To simplify the analysis, a two- phase uncoupled buck with 180 degrees between phases will be discussed here. However, the same approach can be used with any number of phases operating at any phase angle.

A simplified schematic of a two-phase buck is shown in **Fig. 2a**. A two-phase buck has four states of operation. During the first state, the input voltage is connected to phase one, and energy is both being transferred to the output and stored in the inductor L1 (**Fig. 2b**). At the same time, the input side of phase two is connected to ground and the inductor L2 transfers energy to the output. During the second state, the input sides of both phases are connected to ground and both inductors (L1 and L2) transfer energy to the output (**Fig. 2c**). This cycle is repeated over states three and four, the only difference being that phase two is connected to the input while phase one is connected to ground, and then both phases are connected to ground.

Multiphase Uncoupled Inductor

State one of a two-phase uncoupled buck converter covers the phase one on time from t=0 to t=DT_S **Fig. 2b** shows that the output current (I_{OUT}) is the sum of I_1 and I_2, and that during this state the voltage across L1 is V_{IN}-V_{OUT} and that the voltage across L2 is $-V_{OUT}$. Again, using Faraday's law, knowing that dt = DT_S and simplifying:

(Eq. 3) $di_1 = (V_{IN}/L1)\ (1 - D)\ DT_S$

(Eq. 4) $di_2 = (-V_{IN}/L2)\ (D)\ DT_S$

Assuming that L1=L2 = L then

(Eq. 5) $di_{OUT} = di_1 + di_2 = V_{IN}/L\ (1 - 2D)\ DT_S$

These equations show that the current in phase one is increasing as L1 is storing energy, and the current in phase two is decreasing because L2 is sourcing energy.

State two covers the phase-one off time from t=DT_S to t=$T_S/2$. **Fig. 2c** shows that during state two, the output current (I_{OUT}) is still equal to the sum of the currents per phase and that the voltage across both inductors is $-V_{OUT}$. Again using Faraday's law and knowing that dt = $T_S/2$-DT_S = (0.5-D) T_S, we have:

(Eq. 6) $di_1 = (-V_{IN}/L1)\ (0.5 - D)\ DT_S$

(Eq. 7) $di_2 = (-V_{IN}/L2)\ (0.5 - D)\ DT_S$

Assuming L1=L2, then

(Eq. 8) $di_{OUT} = di_1 + di_2 = -V_{IN}/L\ (1 - 2D)\ DT_S$.

During the off time, the current through both inductors is decreasing as both inductors are sourcing energy to the output.

State three covers phase-two on time from t=$T_S/2$ to t=$T_S/2 + DT_S$.

The state three schematic is the same as **Fig. 1b** except that L2 is now connected to the input and L1 is connected to ground. During this state, energy is transferred from the input through phase two to the output and inductor L2 is storing energy. Phase one is connected to ground, and L1 is providing energy to the load. Using the same approach as in state one, the current equations are:

(Eq. 9) $di_2 = (V_{IN}/L2)\ (1 - D)\ DT_S$

(Eq. 10) $di_1 = (-V_{IN}/L1)\ (D)\ DT_S$

Assuming that L1=L2 = L then

(Eq. 11) $di_{OUT} = di_1+di_2= (V_{IN}/L)\ (1 - 2D)\ DT_S$

State four covers phase-two off time from t=$T_S/2 + DT_S$ to t=T_S.

State four is a repeat of state two with both inductors connected to ground and supplying energy to the output.

(Eq. 12) $di_1 = (-V_{IN}/L1)\ (0.5 - D)\ DT_S$

(Eq. 13) $di_2 = (-V_{IN}/L2)\ (0.5 - D)\ DT_S$

Assuming L1 = L2 = L, then

(Eq. 14) $di_{OUT} = di_1 + di_2 = (-V_{IN}/L)\ (1 - 2D)\ DT_S$

It should be noted by the sign of the equations that the output current is increasing in state one and then decreasing in state two, and that this cycle repeats again in states three and four. This implies that the output current ripple occurs at twice the switching frequency. However, the phase-one current is increasing in state one and then decreasing in all three subsequent states (the same is true for phase- two current except the increase occurs in state three) and, therefore, the phase ripple occurs at the switching frequency ($1/T_S$). These waveforms are shown graphically in **Fig. 3.** In summary, for the uncoupled two-phase buck, the current equations are:

(Eq. 15) $di_1 = di_2 = (V_{IN}/L1)\ (1 - D)\ DT_S.$

(Eq. 16) $di_{OUT} = (V_{IN}/L)\ (1 - 2D)\ DT_S.$

Comparing the single-phase (Eq. 1) and uncoupled two-phase (Eqs. 15 and 16) buck regulators, it can be seen that the ripple current per phase remains unchanged (assuming the same L) but that the output ripple has been reduced in the uncoupled two-phase buck regulator.

(Eq. 17) Output Ripple Reduction = $(1 - 2D)/(1 - D)$.

Conversely, to keep the same output ripple, one can reduce the inductance per phase by $(1 - 2D)/(1 - D)$. Reducing the inductance per phase is important because during a load transient, the power supply must be able to respond quickly and does so by either closing all high-side switches or all low-side switches. In either case, it is the size of the inductor (L) that will limit this response.

Multiphase Coupled Inductor

As shown previously, the uncoupled multiphase does an excellent job in addressing the challenges of supplying power for computing applications. The contradictory requirements of both maintaining a minimum output voltage ripple (proportional to di_{OUT}) and allowing for the use of a smaller inductance per phase to enable fast transient response are all addressed. However, as the inductance per phase in the uncoupled multiphase buck is reduced, the ripple current (di) per phase increases. This, in turn, increases the peak current per phase, the switching losses and the I^2R losses, which affect the overall efficiency of the power supply. The goal then is to find a circuit that will enable lower phase ripple while not sacrificing transient response.

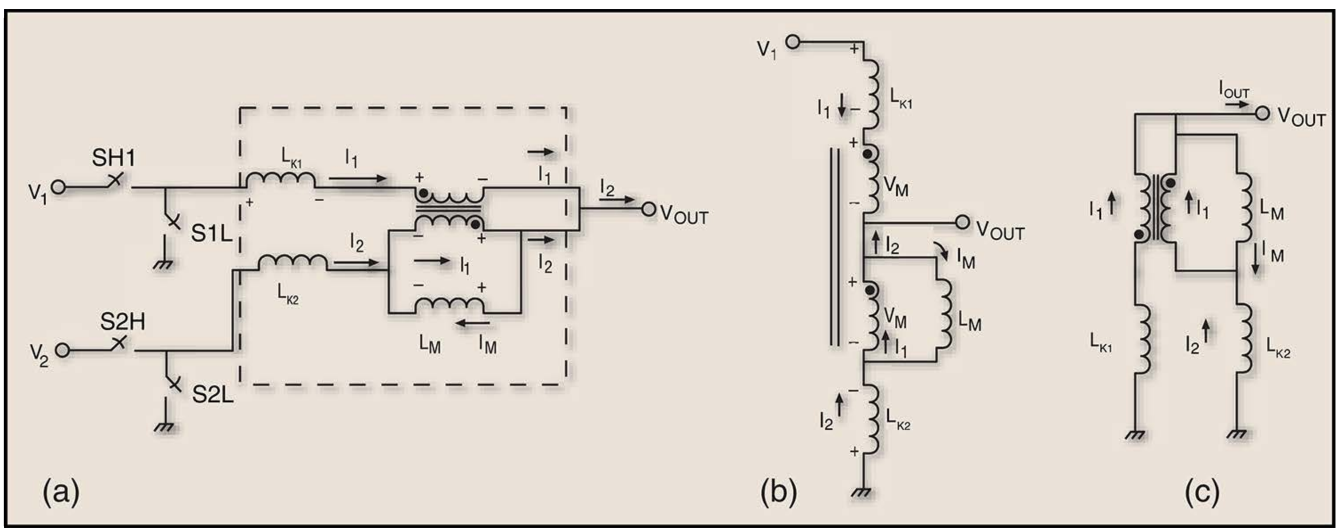

Fig. 4. A simplified schematic of a multiphase coupled buck regulator (a) illustrates the two basic switching actions. In state one (b), S1H and S2L are closed while S1L and S2H are open. The input then sources energy through phase one (L_{K1}) and the output, and phase two (L_{K2}) sources energy to the output. In state two (c), S1L and S2L are closed, and S1H and S2H are open. Thus, both phases source energy to the output. These operations are reversed for states three and four (not shown). Due to the coupling, I_1 and I_2 are not independent.

The coupled inductor approach is similar to the uncoupled version, but instead of using discrete inductors per phase, the inductors are combined on a single core structure. The simplest way to envision a coupled inductor is to think of it as a multiwinding transformer. The standard approach to modeling a transformer is to use an ideal transformer — one that has no losses, perfect coupling and converts voltage and current as a ratio of the turns of each winding, N/N_2 — and to assign a magnetizing inductance (L_M) to one of the windings, which represents the loss associated with the magnetizing current, and a leakage inductance (L_K) to each winding, which represents the loss as a result of imperfect coupling and the energy that is not converted between windings. In the case of the multiphase coupled inductor, each winding will have the same turns (N_1=N_2=N_x); therefore, the voltage and currents on each winding of the ideal transformer will be equal. The magnetizing inductance (L_M) and the sum of the leakage inductances (L_K) can be directly calculated and measured, and therefore provide a good basis for the analysis.

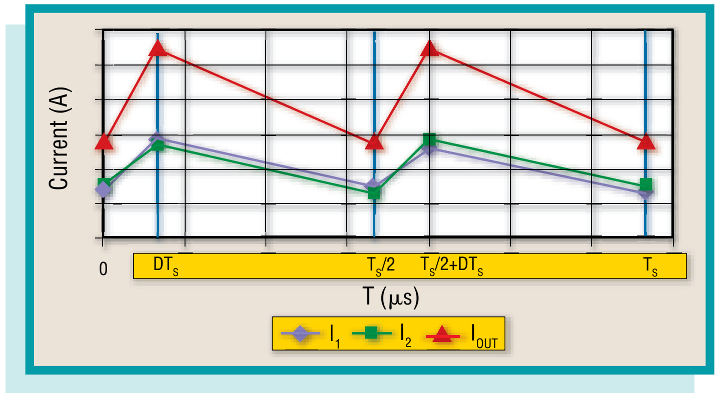

Fig. 5. As the coupling ratio (p) in the two-phase coupled buck regulator increases toward infinity, I_M approaches zero and I_1 and I_2 become equal and in-phase at twice the switching frequency.

Again, to simplify the explanation a two-phase coupled inductor will be analyzed, but the same method can be applied to a greater number of phases. **Fig. 4a** is a simplified schematic of the two-phase coupled topology in which the coupled inductor is represented by two leakage inductances (L_{K1} and L_{K2}), the ideal transformer and magnetizing inductance (L_M). It should be noted that the ideal transformer is connected out of phase with the polarity dots on opposite ends and the current through phase one (I_1) appears as a component of the phase-two current (I_2) due to the coupling of this ideal transformer. For the purpose of this analysis, it is assumed that $L_{K1} = L_{K2} = L_K$ and the following relationships should be noted:

(Eq. 18) $$I_{OUT} = I_1 + I_2$$

(Eq. 19) $$V_{L_{K1}} = L_K\,(di_1/dt)$$

(Eq. 20) $$V_{L_{K2}} = L_K\,(di_2/dt)$$

(Eq. 21) $$V_M = L_M\,(di_M/dt)$$

(Eq. 22) $$I_M = I_1 - I_2$$

or differentiating and using Eq. 21:

(Eq. 23) $$V_M = L_M\,(di_1/dt - di_2/dt)$$

As in the uncoupled version, there are four distinct states and the only difference is that, because of the coupling action, the currents I_1 and I_2 are no longer independent variables.

State one of the multiphase coupled converter covers the phase-one on time from t=0 to t=DT_S. During state one, phase one is connected to the input voltage (S1H closed, S1L open) and phase two is connected to ground (S2H open, S2L closed) and the schematic simplifies to that shown in **Fig. 4b** Looking at **Fig. 4b** and writing the voltage equations around the two current loops, we have:

(Eq. 24) $$V_{L_{K1}} = V_{IN} - V_{OUT} - V_M$$

(Eq. 25) $$V_{L_{K2}} = V_M - V_{OUT}$$

Substituting Eqs. 24 and 25 into Eq. 23, we have:

(Eq. 26) $$V_M = L_M/(L_K + 2L_M)$$

Substituting Eq. 26 back into Eqs. 24 and 25, we have:

(Eq. 27) $$di_1 = (V_{IN}/L_K)\,(1 - D - L_M/(L_K + 2L_M))\,DT_S$$

(Eq. 28) $$di_2 = (V_M/L_K)\,(L_M/(L_K + 2L_M) - D)\,DT_S$$

To simplify the proceeding analysis a ratio, $p = L_M/L_K$ will be used as first shown by Jieli Li, Anthony Stratakos, Aaron Schultz (Volterra) and Charles Sullivan (Dartmouth College) in their paper "Using Coupled Inductors to Enhance Transient Performance of Multi-Phase Buck Converters", and the equations reduce to:

(Eq. 29) $di_1 = (V_{IN}/L_K)((1 + p)/(1 + 2p) - D) DT_S$

(Eq. 30) $di_2 = (V_{IN}/L_K)(p/(1 + 2p) - D) DT_S$

Furthermore, using Eq. 18 we have an output current of:

(Eq. 31) $di_{OUT} = (V_{IN}/L_K)(1 - 2D) DT_S$

State two covers the off time from $t=DT_S$ to $t=T_S/2$. During state two, the input is disconnected entirely (S1H, S2H open) and both windings of the coupled inductor are tied to ground (S1L, S2L closed) and the coupled inductor transfers energy to the load. Looking at **Fig. 4c** and writing the voltage equations around the two current loops we have:

(Eq. 32) $V_{L_{K1}} = -V_{OUT} - V_M$

(Eq. 33) $V_{L_{K2}} = V_M - V_{OUT}$

Substituting Eqs. 32 and 33 into Eq. 23, we have:

(Eq. 34) $V_M = 0$

Substituting Eq. 34 back into Eqs. 32 and 33, we have:

(Eq. 35) $di_1 = di_2 = (-V_{IN}/L_K) DDT_S$

Furthermore, using Eq. 18 we have an output current of:

(Eq. 36) $di_{OUT} = (-V_{IN}/L_K)(1 - 2D) DT_S$

State three covers the phase-two on time from $t=T_S/2$ to $t=T_S/2+DT_S$. The state three schematic is the **Fig. 1b** except that L_{K2} is now connected to the input and L_{K1} is connected to ground. Using the same approach as in state one, it can be shown that:

(Eq. 37) $di = (V_{IN}/L_K)(p/(1 + 2p) - D) DT_S$

(Eq. 38) $di_2 = (V_{IN}/L_K)((1 + p)/(1 + 2p) - D) DT_S$

(Eq. 39) $di_{OUT} = (V_{IN}/L_K)(1 - 2D) DT_S$

State four again covers the off time from $t=T_S/2 + DT_S$ to $t=T_S$. State four is identical to state two, as the input is disconnected and the coupled inductor is tied to ground yielding:

(Eq. 40) $di = di_2 = (-V_{IN}/L_K) DDT_S$

(Eq. 41) $di_{OUT} = (-V_{IN}/L_K)(1 - 2D) DT_S$

Although the coupling ratio does not affect the output ripple, it has a significant affect on the phase current ripple.

Note that the output current is increasing in state one and then decreasing in state two and that this cycle repeats again in states three and four, just as it did in the uncoupled buck. Furthermore, note that coupling ratio (p) does not affect the output current ripple and that if the leakage inductance (L_K) is equal to the value of the individual inductors ($L1 = L2 = L = L_K$) in the uncoupled case, then the output current is unaffected by coupling the inductors. However, it should also be noted that when using coupled inductors, unlike the uncoupled case, the phase currents also change direction twice per cycle implying a phase ripple current at twice the switching frequency (F_S). In reference to **Fig. 5**, as the coupling ratio is increased toward infinity (perfect coupling), the phase currents become equal and in-phase. This is because as L_M goes to infinity, I_M goes to zero, and therefore, I_1 is equal to I_2. Although the coupling ratio (p) does not affect the output ripple, it has a significant affect on the phase current ripple:

(Eq. 42) $di_1 = di_2 = (V_{IN}/L_K)\ ((1 + p)/(1 + 2p) – D)\ DT_S$

Dividing Eq. 42 by the phase ripple in the uncoupled case (Eq. 9) gives:

(Eq. 43) Ripple Reduction (coupled/uncoupled) = ((1 + p)/(1 + 2p) – D)/(1 – D)

As LM goes to infinity (perfect coupling), the coupling ratio goes to infinity and the maximum ripple reduction of (0.5 – D)/(1 – D) is achieved, as shown by Li, Stratakos and Schultz per the previously referenced paper on page 40. Furthermore, as LM goes to zero (no coupling), the coupling ratio goes to zero and the coupled equations revert as expected to the uncoupled case. Fig. 6 shows the ripple reduction as the coupling ratio is varied for different duty cycles. It should be noted that for all practical purposes a coupling ratio in excess of three or four allows for most of there duction to be gained. As can be seen for a 10%, duty cycle, the best reduction is 44%, but as the duty cycle is increased to 50%, it is possible to achieve complete ripple cancellation. Conversely, it is possible to reduce the leakage inductance (L_{K1} and L_{K2}) and still have the same ripple per phase as in the uncoupled version using higher values of L1 and L 2. This is significant because during a transient condition (S1H, S2H closed or S1L, S2L closed), the coupling ratio falls out of the Eqs. 35 and 40, and as a result the transient response time will be based only on the value of L_K.

The Final Analysis

It is apparent from this analysis that the uncoupled multiphase buck allows for reduced output ripple when compared to the single-phasebuck. Conversely, if lower values of inductance are used, it can provide the same output ripple with a faster transient response.The penalty in the uncoupled multiphasebuck is the increased ripple perphase and associated losses. The coupled multiphase buck solves this by allowing for reduced phase ripple for the same output ripple, or conversely, if lower values of leakage inductance are used, one can achieve even faster transient response without increasing the phase ripple.

It is possible to take this analysis further by showing that a lower effective transient inductance allows for a reduction in bulk capacitors offering a significant cost reduction, and that with the coupled solution, it is possible to slow down the switching frequency (and there by reduce switching losses) and still maintain acceptable output and phase ripple and adequate transient response while increasing the overall efficiency of the power supply.

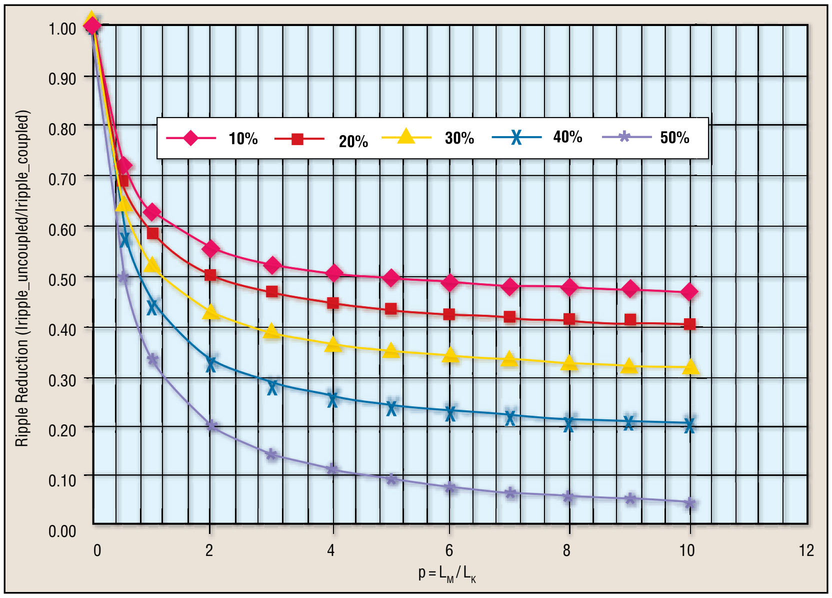

Fig. 6. Ripple reduction in a two-phase coupled inductor buck regulator varies with duty cycle and coupling ratio (p). As magnetizing inductance L_M of the coupled inductor approaches zero, the coupling ratio approaches zero and there is no ripple reduction. As L_M approaches infinity, the coupling ratio approaches infi nity and the ripple reduction approaches the maximum reduction.

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